FHWA Workshop over the Web for Travel Model Development Session 7 Homework

Static Equilibrium Highway Assignment

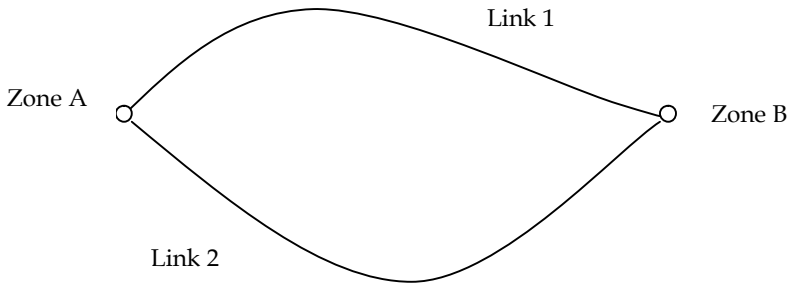
Total Time Required: 30 Minutes

In this exercise, we will perform a static equilibrium highway assignment on a really simple network. The assignment will be performed using a standard BPR formula as the volume-delay function.

Background

Consider the network shown in Figure 1, which may well be the simplest you have ever seen.

Figure 1. Example Network



In the a.m. peak period, there are 4000 trips, and they all have Zone A as the origin and Zone B as the destination. There are two possible paths between A and B, via Link 1 and via Link 2. Table 1 shows the characteristics of the two links.

Table 1. Link Characteristics

Link	Length (miles)	Free flow speed (mph)	Capacity (vehicles in period)	Free flow travel time
1	10.0	50	2500	
2	10.0	40	2000	

We will perform a most of the functions of a six-iteration static equilibrium highway assignment for this network and "trip table." Remember, in each iteration an all-or-nothing assignment is performed, and the results of all iterations are averaged using weights. The standard BPR formula, with the original parameters, is used:

$$T_k = T_{0k} [1 + a (v_k/c_k)^b]$$

where:

- T_k = travel time on link k (k = 1, 2)
- T_{0k} = free flow travel time on link k
- v_k =volume on link k
- c_k = capacity on link k

We will produce the usual outputs of the highway assignment, namely, the volumes and travel times on Links 1 and 2.

We will not compute the weights for the six iterations; they have been provided in advance. In an actual equilibrium assignment, the weights would be computed by the modeling software using an optimization procedure such as Frank-Wolfe.

Questions

1. Calculate the free flow travel times for the two links.

2. Open the spreadsheet **Homework session 7.xls**. The first three iterations of the assignment have been done. Examine the calculations of the weighted average volumes (Columns J and M) and travel time, from the BPR formula (Columns H, K, and N). The descriptions of the calculations for iterations 1-3 are shown below.

   Iteration 1 – The all-or-nothing assignment assigns all 4000 trips to the link with the shorter travel time. In this case, Link 1 has a lower travel time, and so all 4000 trips are assigned to it. There are no previous iterations to average.

   The BPR formula computes the new travel time for each link, in cells H4 and H5. For example, for Link 1:

   $$T_1 = T_{01} [1 + 0.15 (v_1/c_1)^4]$$

   $$= 12 [1 + 0.15 (4000/2500)^4]$$

   $$= 23.8$$

   Iteration 2 – The travel time is lower on Link 2, and so the all-or-nothing assignment assigns all 4000 trips to Link 2 (Column I). With the weight of 0.500 for iteration 1 and 0.200 for iteration 2, the weighted average volumes are 2857 for

Link 1 and 1143 for Link 2 (Column J). The new travel times, computed using the BPR formula (Column K), are 15.1 minutes for Link 1 and 15.2 minutes for Link 2.

Iteration 3 – The travel time is lower on Link 1, and so the all-or-nothing assignment assigns all 4000 trips to Link 1 (Column L). With the combined weight of 0.700 for iterations 1 and 2 and 0.200 for iteration 3, the weighted average volumes are 3111 for Link 1 and 889 for Link 2 (Column M). The new travel times, computed using the BPR formula (Column K), are 16.3 minutes for Link 1 and 15.1 minutes for Link 2.

3. Fill in the volumes for the all or-nothing assignments for iteration 4 (Column O), iteration 5 (Column R), and iteration 6 (Column U). The cells to fill in are shown in light purple (Excel calls this color pink, but it looks purple to me). When you fill in the volume for an iteration, the weighted volumes and travel times (yellow cells) will recalculate. These recalculated travel times will be your basis for determining the all-or-nothing assignment results for the next iteration.

4. What are the final volumes for each link after iteration 6? What are the final travel times and speeds?

5. Do you feel that this assignment has converged based on the definition of equilibrium?